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Topological defects in condensed matter systems

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Abstract : Topological defects arise in many condensed matter systems and have been extensively studied theoretically as well as experimentally This talk gives an introduction to this subject, with an emphasis on topological defects in liquid crystal systems Some recent experiments with liquid cyrstal defects are discussed which have important implications for theories of cosmic defects in the early universe.

Keywords : Topological defects, liquid crystals, phase transition

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1. Introduction

We present a brief review of topological defects in condensed matter systems. In the following, we first explain general properties of topological defects. We explain the concept of topological defects in detail using a simple example of a string defect. Then we explain how these defects are produced in phase transitions. We also describe certain recent experiments with liquid crystal defects which provide experimental verifications of theories of defects formation. It is important to emphasize that the subject of topological defects is highly interdisciplinary. Most of what we discuss below applies to condensed matter systems as well as to other fields where topological defects arise, for example in particle physics (with appropriate translation of terminology).

It is simple to explain the concept of topological defects using condensed matter systems having phase transitions. Consider a system which can exist in two different phases, for example steam and water, superconductors with normal and superconducting phases *etc*. Generally, one defines an *order parameter*, which takes different values in different phases. Existence of topological defects crucially relies on the nature of this order parameter depending on which, several kinds of topological defects can exist.

Point defects (monopoles) :

These are tiny point like regions of one phase embedded in the other phase. Like a tiny drop of water in steam (but very different in nature as we will explain below).

String defects :

These are thin tube like regions of one phase embedded in the other phase.

Domain walls :

These are sheet like regions of one phase embedded in the other phase.

Strings (domain walls) are either closed loops (surfaces), or they end at the boundaries of the condensed matter system (in the context of particle theory models of the early universe, they can be infinitely large). Their structure shows the origin of the term 'defect'. A defect represents a localized region of one phase embedded in the background of a different phase. These defects are topological because their existence originates from topological considerations.

A given property of a system is said to be of topological nature if smooth deformations (continuous changes) do not change that property. Topological defects are stable against smooth deformations (continuous changes) in the system. This is where, for example, a topological point defect differs from the example of a water droplet embedded in steam. Local heating can easily convert the water droplet to steam so that there is no water left anywhere. On the other hand, local heating, or other local deformations can not get rid of a topological point defect.

Kinks, or domain walls are the simplest topological defects. A nontrivial, and physically more interesting example of topological defects is a string defect We consider string defect in a specific system.

2. String defects in superfluid ^{4}He

An example of string defect is vortices in superfluid ^{4}He. The order parameter which describes the superfluid phase is a complex scalar field ψ. In the normal phase $\psi = 0$ while in the superfluid phase $|\psi| \neq 0$.

The free energy density F is of the form

$$F = K|\nabla\psi|^2 - \alpha|\psi|^2 + \beta|\psi|^4, \qquad (1)$$

α is negative for temperatures $T > T_c$, and becomes positive for $T < T_c$. Plot of F (for spatially uniform ψ) is shown in Figure 1, here $\psi = \psi_1 + i\psi_2$.

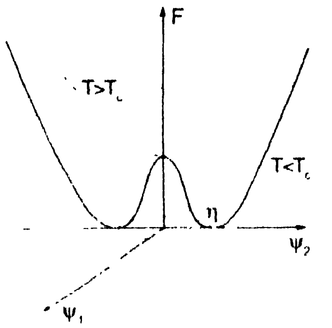

Figure 1. Free energy for superfluid ^{4}He

We see that for $T > T_c$, F is minimized for $\psi = 0$ while in the superfluid phase ($T < T_c$) F is minimized for $|\psi| \equiv \eta = \sqrt{2\alpha/\beta}$. Clearly this does not fix the phase θ of ψ (which can vary spatially in a phase transition). This remaining degree of freedom spans what is called as the order parameter space V, which is a circle S^1 in this case. Of course, any spatial variation of ψ will cost energy due to the gradient term in the expression for F.

Consider now a region of superfluid with the distribution of θ on (and nearby) a closed path L in physical space such that θ changes by 2π as we go around the path L. As $\Delta\theta$ can only change by an integer multiple of 2π around L, its value can not change if we make a small deformation of L. $\Delta\theta$ is thus an example of a topological invariant which does not change under continuous deformations. It then follows that one can continuously shrink L down to a point while $\Delta\theta$ around L remains 2π (as long as we do not cross any region of $\psi = 0$ where θ becomes undefined). When L shrinks to a point then ψ must go to zero there to maintain finite gradient energy density (eq. (1)). Thus, we conclude that the original loop L must enclose at least one point where ψ vanishes identically. As L can be shrunk on any surface whose boundary is L, we conclude that L encloses a line like region where $\psi = 0$. Since vanishing of ψ implies normal phase of ^{4}He, one obtains a string like region of normal phase embedded in the superfluid phase of ^{4}He. This is the vortex in superfluid ^{4}He.

Note that in this example, existence of string was related to the fact that there was a closed curve in the order parameter space V (which was S^1 in the above example), which could not be shrunk to a point within V. $\Delta\theta$ can change by $2n\pi$ around L, with each different value of integer n (the winding number of loop in V) corresponding to a topologically distinct string. When there are such non-trivial loops which can not be smoothly shrunk to a point, one says that the 1st homotopy group of V, $\pi_1(V)$ is non-trivial. These ideas are easily generalized to other homotopy groups by considering higher dimensional surfaces in V which can not be smoothly shrunk to point. For example, $\pi_2(V)$ ($\pi_3(V)$) non-trivial means that there are closed 2-dimensional surfaces (3-dimensional surfaces) in V which can not be smoothly shrunk to a point. $\pi_0(V)$ non-trivial means that V is disconnected.

In general, defects (in 3 space dimensions) are classified in the following manner. For $\pi_n(V) \neq 1$ with n being either 0, 1, 2 or 3, one gets a domain wall defect, a string defect, a monopole, or texture (Skyrmion), respectively. [One actually needs to consider what is called as free homotopy as opposed to based homotopy which is used in defining $\pi_n(V)$. We will not go in these details. For details see [1].]

3. Defect formation in phase transitions

During any phase transition at finite temperature, defects will be produced due to thermal fluctuations. Apart from this thermal production, there is a *non-equilibrium process* which dominates at low temperatures. This process of defect formation is generally known as the Kibble mechanism and arises due to a sort of domain formation after the phase transition with defects forming at the junctions of these domains [2]. This mechanism was originally proposed by Kibble in the context of cosmic defect formation in the early universe. It was subsequently realized [3] that it applies to condensed matter defects as well, for example defects in superconductors, superfluid helium, liquid crystals *etc.*

We will describe this mechanism by considering a first order phase transition (this will be the case for liquid crystal defect formation, as we discuss later). Same ideas apply for 2nd order transition also. Let us take the order parameter to be again a complex scalar ψ with the order parameter space being a circle. We have seen earlier that there are string defects in this case which are characterized by non-trivial winding of the phase θ of ψ. Note that the plot of F in Figure 1 corresponds to a second order phase transition. First order transition will happen if there is another local minimum of F at $\psi = 0$.

As the system is supercooled in the metastable phase with $\psi = 0$, the phase transition proceeds by nucleation of critical bubbles of the low temperature phase. These bubbles grow and eventually coalesce, completing the phase transition. The magnitude of ψ is fixed in each bubble, but the phase θ varies randomly from one bubble to another (being roughly uniform inside a given bubble). Eventually, when bubbles coalesce, one is left with random variation of θ from one region to another, giving rise to a sort of domain structure in space. Occasionally, one will get regions where θ winds non-trivially around a point, leading to a defect there.

Same thing happens for a 2nd order transition with θ varying randomly beyond the correlation length. Using random variation of θ from one domain to another, one can estimate probability of defect formation per bubble (per correlation volume for 2nd order transition case) to be 1/4 (for two space dimensions). It is important to realize that this probability (per domain) only depends on the space dimensions and the topology of the order parameter. The details of the system are only relevant in determining the bubble (domain) size *etc*.

4. Experimental studies of defect formation in liquid crystals

Liquid crystals are orientationally ordered liquids formed by elongated (rod like) or flat (disk like) molecules. There are variety of liquid crystalline phases and topological defects associated with them [4]. We will only discuss nematic liquid crystals here (NLC). In the nematic phase, the molecules are locally aligned while in the high temperature isotropic phase the molecules are randomly oriented. The order parameter gives the strength of local ordering in the nematic phase. Its orientation is given by the *director* which describes the local axis of the orientational order. Opposite orientations of the director are identified, hence the order parameter space is $S^2/Z_2 \equiv RP^2$ (the projective plane). There are string defects here since $\pi_1(S^2/Z_2) = Z_2$. These correspond to winding of the director by angle π around the string defect. Note that the director can wind by π here around a closed loop since opposite orientations of the director are identified. There are other defects in the nematic phase, for example point defects (monopoles), and textures, see [4].

Experimental observations of defects in liquid crystals have a long history. These defects are easily observed under optical microscopes. Using cross-polarizers, one can also determine the winding number of these defects. The first experimental study of the evolution of string network formed in a phase transition, which was extensively studied theoretically for the case of cosmic strings, was carried out by Chuang *et al.* [5]. They used a pressure cell to carry out the isotropic-nematic transition and observed the scaling behavior of string network.

Kibble mechanism for defect production leads to two important predictions. One is about the density of defects per domain (bubble), and the other relates to correlations between defects and antidefects. Important thing is that these are universal predictions in the sense that they depend only on the space dimension and the order parameter space. Thus, even though the Kibble mechanism was proposed for cosmic defects in the context of the early universe, one can check these predictions in the laboratory in condensed matter systems. We now discuss two experiments where this has been done by studying defect production in nematic liquid crystals.

Density of defects :

An experimental verification of the prediction of defect density from Kibble mechanism was carried out in a work reported in Ref. [6]. This was achieved by counting the density of strings produced in a nematic-isotropic phase transition. This study was done by placing a drop of NLC K15 on a clean, untreated microscope slide. The drop was heated by an illuminator. Following a slow reduction of light intensity we were able to obtain good images of bubble formation and evolution as the drop cooled through the isotropic-nematic phase transition at 35.3°C. Figure 2 shows a typical sequence of images where one can see the beginning of the phase transition by bubble nucleation. Subsequently, bubbles coalesce and lead to the formation of string network. String network coarsens as strings shrink. Observations of strings produced per bubble is found to be in good agreement with the theoretical predictions [6].

Figure 2. Various stages of defect formation. Starting from left, first figure shows nucleation of bubbles of broken symmetry phase (nematic phase in the liquid crystals). Second picture shows dense network of string defects formed due to coalescence of bubbles. Third picture shows how the string network coarsens as strings shrink and string loops decay.

Defect-antidefect correlations :

Kibble mechanism predicts the existence of correlations between defects and antidefects formed during a phase transition. In a given region containing N defects and antidefects on the average, if defects and antidefects were uncorrelated then one expects that the net defect number $\Delta N \equiv N_d - N_{\bar{d}}$ will be distributed about zero with a width ~ $N^{1/2}$. The Kibble mechanism predicts specific defect correlations so that the width of the distribution of ΔN varies as $N^{1/4}$.

The experimental verification of this prediction has been carried out in a work reported in Ref. [7]. Defects and antidefects are identified using cross-polarizers and by using topological arguments. Figure 3 shows a photograph of a point defect

Figure 3. Observations of liquid crystal defects using cross-polarizers. Crossings of brushes correspond to defects with windings ± 1.

network observed using cross-polarizers. Crossing of dark brushes corresponds to defects with the director winding by 2π around the defect.

After identifying the windings of defects, distributions of ΔN are plotted for different average values of $N \equiv N_d + N_{\bar{d}}$, and the widths of these distributions are determined. We check the following scaling relation

$$\sigma = C N^{\nu}. \tag{2}$$

In the absence of any correlations between defects and antidefects, ν should be 1/2. Kibble mechanism predicts $\nu = 1/4$. Prediction of C from the Kibble mechanism is less constrained (it depends on the shape of elementary domains *etc.*). One expects that $C = 0.57$ or 0.71.

Our experimental results lead to the following values for ν and C.

$$\nu = 0.26 \pm 0.11, \quad C = 0.76 \pm 0.21. \tag{3}$$

These experimental results are in very good agreement with the predictions of the Kibble mechanism and clearly rule out uncorrelated defect-antidefect production.

5. Conclusions

The interdisciplinary nature of the subject of topological defects has led to a valuable interplay of ideas from various branches of physics. It has led to the remarkable possibility of checking predictions relating to cosmic defects in condensed matter systems. Topological defects can be experimentally studied using rather simple experimental setups such as nematic liquid crystal experiments described here. This opens up the opportunity for doing table top experiments which can lead to important checks on the theories of defect formation.

I would like to thank the organizers and other participants for a great conference.

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